

# Cluster state generation with atomic ensembles via the dipole blockade mechanism

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We present a new scheme for cluster states generation based on atomic ensembles and the dipole blockade mechanism. The protocol requires identical single photon sources, one ensemble per physical qubit, and regular photodetectors. The general entangling procedure is presented, as well as a procedure that generates  $Q$ -qubit GHZ states with probability  $p \sim \eta^{Q/2}$ , where  $\eta$  is the combined detection and source efficiency. This is significantly more efficient than any known robust probabilistic entangling operation. The GHZ states form the basic building block for universal cluster states — a resource for the one-way quantum computer.

*Introduction.* The construction of a quantum computer is an important goal of modern physics, and one possible implementation is via atomic ensembles: The quantum state of the ensemble can be coherently manipulated with light, and the decoherence of the quantum information can be highly suppressed [1, 2, 3, 4, 5]. It is therefore possible to define a “good” qubit in an atomic ensemble, and the question remains how to implement the entangling operations between the qubits that enable universal quantum computation. It suffices to create a large entangled multi-qubit resource — the cluster state — after which the entire computation proceeds via single-qubit measurements [6, 7]. Here, we show how to create these cluster states using the dipole blockade mechanism. The protocol requires identical single photon sources, one ensemble per physical qubit, and regular photodetectors. We present a general entangling procedure, as well as a procedure that generates  $Q$ -qubit GHZ states with probability  $p \sim \eta^{Q/2}$ , where  $\eta$  is the combined detection and source efficiency. This is significantly more efficient than any known robust probabilistic entangling operation [8, 9]. The GHZ states form the basic building block for universal cluster states.

The physical mechanism that is central to our proposal is the dipole blockade mechanism in the atomic ensemble: An optical pulse resonant with a transition to a Rydberg state will create a Rydberg atom with a very large dipole moment. When the atoms in the ensemble are sufficiently close, the dipole interaction between the Rydberg atom and the other atoms will cause a shift in the Rydberg transition energy of the other atoms. Therefore, the optical pulse becomes off-resonant with the other atoms, and the ensemble is transparent to the pulse. This mechanism prevents populating states of atomic ensembles with two or more atoms excited to the Rydberg level [2, 10].

The range and quality of the dipole interaction has been studied extensively: Walker and Saffman analyzed the primary errors that enter the blockade process [11, 12]. For Rubidium atoms with principal quantum number  $n = 70$ , the blockade energy shift is approximately 1 MHz. Hence, a strong and reliable blockade is possible for two atoms with separation up to  $\sim 10 \mu\text{m}$  [11]. Moreover, decoherence associated with spontaneous

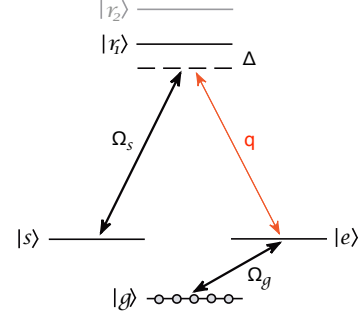


FIG. 1: Relevant atomic level structure with allowed atomic transitions. States  $|g\rangle$ ,  $|e\rangle$  and  $|s\rangle$  can be realized by the electronic low-lying states of alkali atoms. The transition between states  $|g\rangle$  and  $|s\rangle$  is always dipole-forbidden. The state  $|g\rangle$  is coupled to the state  $|e\rangle$  through a classical field  $\Omega_g$ . A second classical field  $\Omega_s$  is at resonance with the transition between the highly excited Rydberg level  $|r_1\rangle$  and the state  $|s\rangle$ . States  $|e\rangle$  and  $|r_1\rangle$  are coupled via a quantum field.  $|r_2\rangle$  is an auxiliary Rydberg level used in single-qubit operations.

emission from long-lived Rydberg states can be quite low ( $\sim 1 \text{ ms}$ ). The dipole blockade mechanism can be used to build fast quantum gates, i.e., a two qubit phase gate [13, 14, 15]. The long-range dipole-dipole interaction between atoms can be employed to realize a universal phase gate between pairs of a single-photon pulses [16, 17, 18].

*Protocol.* Each qubit is represented by a spatially separated atomic ensemble. The atoms in each ensemble have three lower, long lived energy states  $|g\rangle$ ,  $|e\rangle$  and  $|s\rangle$  (see Fig. 1). The qubit states in an ensemble of  $N$  atoms are:

$$|0\rangle_L \equiv |g\rangle = |g_1, g_2, \dots, g_N\rangle, \quad (1)$$

$$|1\rangle_L \equiv |s\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^N |g_1, g_2, \dots, s_j, \dots, g_N\rangle. \quad (2)$$

The states  $|e\rangle$  and  $|r_1\rangle$  participate in the interaction part of the scheme. Levels  $|g\rangle$  and  $|s\rangle$  play the role of the storage states. Single-qubit operations are realized by means of classical optical pulses and the dipole blockade mechanism. An arbitrary phase gate  $\Phi(\phi) = \exp(-i\phi Z/2)$  is

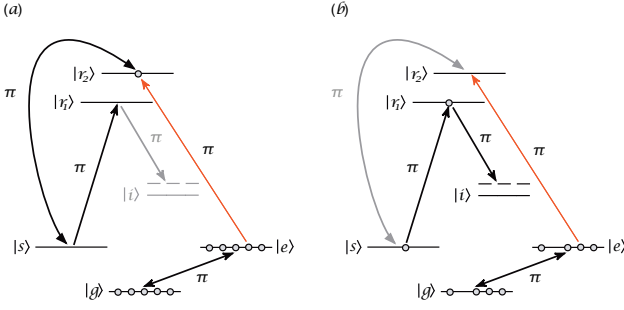


FIG. 2: The bit flip operation ( $X$ ) and the Hadamard gate ( $H$ ). First, two  $\pi$ -pulses are applied to the transitions  $|g\rangle \leftrightarrow |e\rangle$  and  $|s\rangle \leftrightarrow |r_1\rangle$ . Then we send a pulse resonant with the transition  $|e\rangle \leftrightarrow |r_2\rangle$ . Finally, two pulses couple levels  $|r_1\rangle$  and  $|e\rangle$  (STIRAP through the level  $|i\rangle$ ) and levels  $|r_2\rangle$  and  $|s\rangle$ . We apply the same sequence of classical pulses in both cases. (a)  $|0\rangle_L \rightarrow |1\rangle_L$  After a first two pulses no atom is present in the state  $|r_1\rangle$ . Therefore, one atom may be excited to the state  $|r_2\rangle$  and then transferred to the state  $|s\rangle \equiv |1\rangle_L$ . (b)  $|1\rangle_L \rightarrow |0\rangle_L$  Now the presence of one atom in the state  $|r_1\rangle$  blocks the excitation of another atom to the state  $|r_2\rangle$ . The STIRAP procedure transfers a single atom from state  $|r_1\rangle$  to state  $|e\rangle$ . In this setup, the Hadamard gate can be realized by a  $\pi/2$ -pulse between Rydberg states  $|r_1\rangle$  and  $|r_2\rangle$ . A single excitation can be transferred to the storage states  $|g\rangle$  and  $|s\rangle$ . More details about single-qubit operations can be found in Ref. [19].

realized by a detuned optical pulse applied to the transition between  $|s\rangle$  and an auxiliary level  $|a\rangle$  (not shown). The Pauli  $X$  operation (the bit flip) and the Hadamard gate  $H$  are shown in Fig. 2. The gates  $\Phi(\phi)$ ,  $X$ , and  $H$  generate all single-qubit operations. The readout of a qubit is based on resonance fluorescence. If the measurement gives no fluorescence photons, the qubit is in  $|0\rangle_L$ . Otherwise, the state of the qubit is projected onto  $|1\rangle_L$ .

The entangling operation is constructed as follows: Two atomic ensembles are placed in the arms of a Mach-Zehnder interferometer (see Fig. 3). Initially, we prepare each ensemble  $A$  and  $B$  in the state  $|\phi_{A,B}\rangle = |e\rangle \equiv |e_1, e_2, \dots, e_N\rangle$ . Two indistinguishable photons enter each input mode of the interferometer, and due to the Hong-Ou-Mandel (HOM) effect, after the first beam splitter ( $BS_1$ ) the two photons are in the state  $|\phi_{light}\rangle = |11\rangle \xrightarrow{BS_1} \frac{i}{\sqrt{2}}(|02\rangle + |20\rangle)$  where  $|0\rangle$  and  $|2\rangle$  denote the vacuum and a two-photon state, respectively. After  $BS_1$  the photons interact with the atomic ensembles: one and only one atom in the ensemble is excited by one of the photons to a Rydberg state  $|r_1\rangle$ , and the absorption of the second photon is prohibited by the dipole blockade mechanism. The total state of a ensemble-light system after interaction is given by:

$$|\phi_{int}\rangle = \frac{i}{\sqrt{2}}(|er_1\rangle|01\rangle + |r_1e\rangle|10\rangle). \quad (3)$$

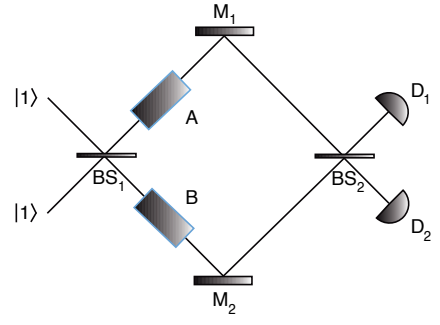


FIG. 3: Diagram of the protocol. We send an entangled pair of photons in the state  $|\phi_{light}\rangle = \frac{i}{\sqrt{2}}(|02\rangle + |20\rangle)$  into ensembles  $A$  and  $B$ . The photons interact with atomic vapours: one and only one alkali atom in the ensemble is excited by one of the photons to a Rydberg state  $|r_1\rangle$ . Absorption of the second photon is prohibited by the dipole blockade mechanism. After  $BS_2$ , a state of ensembles-light system has the following form  $|\phi_{out}\rangle = \frac{i}{\sqrt{2}}(|\psi^+\rangle|01\rangle + |\psi^-\rangle|10\rangle)$ , where  $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|r_1e\rangle \pm i|er_1\rangle)$ . Detection of a single photon will leave the atomic ensembles entangled.

After the second beam splitter ( $BS_2$ ), the total state is

$$|\phi_{out}\rangle = \frac{i}{\sqrt{2}}(|\psi^+\rangle|01\rangle + |\psi^-\rangle|10\rangle), \quad (4)$$

where  $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|r_1e\rangle \pm i|er_1\rangle)$ . Conditional on a single photodetector click, the ensembles are projected onto a maximally entangled state. After establishing entanglement, the qubits are transferred to their computational basis states  $|0\rangle_L \equiv |g\rangle$  and  $|1\rangle_L \equiv |s\rangle$  by classical optical pulses  $\Omega_g$  and  $\Omega_s$ . This means that ideally every run of the procedure would give an entangled state of ensembles with success probability  $p = \eta$ , where  $\eta$  is the combined detection and source efficiency. This is a significant improvement compared to the success probability  $p = \eta^2/2$  of the double heralding protocol in Ref. [8].

**GHZ and cluster states.** The entangling operation can be used to efficiently create arbitrary cluster states, including universal resource states for quantum computing. However, a modification of the entangling procedure yields an even more dramatic improvement in the efficiency of cluster state generation. By arranging the ensembles in a four-mode interferometer as shown in Fig. 4, the detection of two photons will create a four-qubit GHZ state in a single shot. Moreover, since only two photons are detected, the protocol is relatively insensitive to detector losses. The success probability is  $p = \eta^2/2$ . Higher GHZ states can be created by a straightforward extension. A subsequent cluster states are generated with probability  $p = \eta^{Q/2}(Q-2)/2^{Q-2}$  where  $Q = 4, 6, \dots$  is the number of the qubits.

The efficiently generated large GHZ states may serve as a building block for creating arbitrary cluster states. By entangling small clusters with the above entangling procedure, large cluster states can be constructed. A single

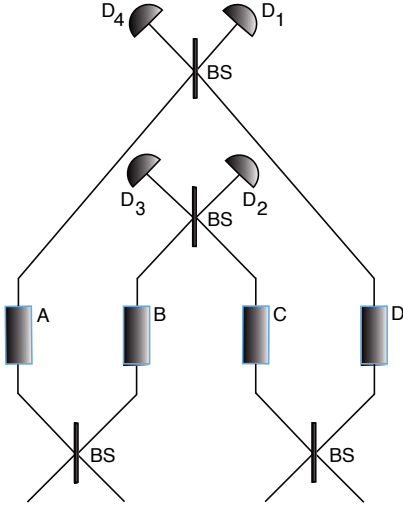


FIG. 4: The scheme for creating a 4-qubit GHZ state. The four ensembles  $A$ ,  $B$ ,  $C$  and  $D$  are prepared in the state  $|\phi_{ABCD}\rangle = |eeee\rangle$ . Four indistinguishable photons are sent into the beam splitters. The interaction of photons with the atomic vapours is followed by the beamsplitters and four photodetectors. Conditional on photodetector clicks at the photodetector  $(D_1, D_2)$ ,  $(D_1, D_3)$ ,  $(D_4, D_2)$  or  $(D_4, D_3)$ , a state of the four qubits is projected onto the 4-qubit GHZ state (up to phase correcting operations) with success probability  $p = \eta^2/2$ .

photon applied to a pair of qubits (each from two different 4-qubit cluster states) followed by a single photodetector click creates a 8-qubit cluster state with success probability  $p = \eta'/8$ . This procedure can be repeated in an efficient manner [20]. In case of failure, the two qubits that participated in linking are measured in the computational basis, and the rest of the cluster state is recycled.

*Errors, decoherence mechanisms and fidelity.* The dominant errors and decoherence mechanisms are the coincident event in the HOM effect, the spontaneous emission rate of the Rydberg state, the black-body transfer rate (to other Rydberg states), the atomic collision rate, the doubly-excited Rydberg states and singly-excited states outside the desired two-level system, no absorption event, and the inefficiency and the dark count rate of the photodetectors. These errors are discussed in the next section of article. Considering the time scale of the protocol, the entangling procedure is mostly affected by the no absorption event and inefficiency of the photodetectors (we assume that the coincident event rate in the HOM effect is negligible). In presence of above noise and decoherence mechanisms, the final state of the system conditional on a single photodetector click is given by

$$\rho_{fin} = (1 - 2\varepsilon) |\psi^\pm\rangle\langle\psi^\pm| + 2\varepsilon \rho_{noise} + \mathcal{O}(\varepsilon^2), \quad (5)$$

where  $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|sg\rangle \pm |gs\rangle)$  and  $\varepsilon = 1 - P_{abs}$ .  $\rho_{noise}$  denotes the unwanted terms in the state of the two en-

sembles. The source efficiency does not affect the fidelity of the final state, just lowers the success probability. After taking into account all dominant error mechanism, the fidelity of the prepared entangled state is  $F = \langle\psi^\pm|\rho_{fin}|\psi^\pm\rangle \cong 0.982$ , which is close to current fault-tolerant thresholds [21].

*Experimental implementation.* Let us analyze in more detail mentioned dominant error and decoherence mechanisms. First, consider the coincident events in the HOM effect. The single indistinguishable photons that recombine at the first beam splitter ( $BS_1$ ) can be generated by means of a spontaneous parametric down-conversion (SPDC) process. In general, successful generation of the entangled state of light depends on the proper setup, where both photons recombine at  $BS_1$  at the same time. In a recent experiment, the coincident event in the HOM effect happens with a rate of 1500 counts/s [22]. We assume that the rate of coincident events is negligible comparing to the time scale of the protocol which is  $t \cong 5\mu s$ .

Assume that an atomic vapour consists of 300  $^{87}\text{Rb}$  atoms placed in the far-off-resonant optical trap or magneto-optical trap (MOT). The atomic levels  $|g\rangle$ ,  $|e\rangle$  and  $|r\rangle$  may correspond to  $5S_{1/2}$ ,  $5P_{3/2}$  and  $43D_{5/2}$  or  $58D_{3/2}$ , respectively. State  $|s\rangle$  corresponds to the long lived, lower energy level. The spatial distribution of an atomic cloud is a quasi one-dimensional ensemble with probability density  $P(z) = (2\pi\sigma^2)^{-1/2}\exp(-z^2/2\sigma^2)$  where  $\sigma = 3.0\mu m$ . Atomic vapours described with quasi one-dimensional probability density have been demonstrated experimentally [10].

When a protocol is based on a quantum optical system, its performance is limited by the inefficiency and the dark count rate of the photodetectors. The dark count rate of the modern photodetector can be as low as 20 Hz and efficiency reaches  $\eta \approx 30\%$  for wavelengths around 480 nm. The probability of the dark count is  $P_{dc} = 1 - \exp(-\gamma_{dc}t/p_{success}) \cong 5 \cdot 10^{-4}$ . In general, the probability of the dark count is negligible for  $p_{success} > \gamma_{dc}t$ .

Since the length of the atomic ensemble needs to be of order of  $\mu m$ , the most important source of errors is the lack of absorption event. The probability of an absorption of a single photon by an atomic ensemble is given by  $P_{abs} \cong 1 - e^{-N_i\sigma_0/A}$  with  $N_i$  the number of atoms in the interaction region,  $\sigma_0 = 3\lambda^2/(2\pi)$  and  $A = \pi w_0^2$  [23]. With  $\lambda_{43D} = 485.766\text{ nm}$ ,  $\lambda_{58D} = 485.081\text{ nm}$  and  $w_0 \approx \pi\lambda$ , the probability of an absorption for both Rydberg states is  $P_{abs} \cong 0.989$ .

The probability of doubly-excited Rydberg states (absorption of both photons by an ensemble) depends on the quality of the dipole blockade and is given by the following expression  $P_2 = (N - 1)g_N^2/2NB^2$ , where  $g_N = \sqrt{N}g_0$  and  $B$  is the mean blockade shift [11]. For  $43D_{5/2}$  and  $58D_{3/2}$  mean blockade shift  $B = 0.25\text{ MHz}$  and  $B = 2.9\text{ MHz}$  in a trap with  $\sigma = 3.0\mu m$ , respectively [11]. Hence, the probability of doubly-excited

states for  $43D_{5/2}$  level is  $P_2 \cong 0.26$  and for  $58D_{3/2}$  level is  $P_2 \cong 0.57 \cdot 10^{-3}$ . The probability of doubly-excited states and singly-excited states outside the desired two-level system are similar. Above probabilities are given for the worst case scenario when the separation of atoms is maximal and the dipole-dipole interaction is of the weakest, van der Waals type.

The spontaneous emission from the Rydberg state and the black-body transfer (to other Rydberg states) occur with rate of order  $10^3$  Hz, and is negligible, since after successful entanglement preparation the state of ensemble is stored in the long lived atomic states  $|g\rangle$  and  $|s\rangle$ . Exact values of these rates are given in Ref. [22, 24]. The atomic collision rate is given by  $\tau_{col}^{-1} \approx n\sigma_{col}/\sqrt{M/3k_B T}$  with  $n$  the number density of atoms,  $\sigma_{col}$  the collisional cross section ( $\sim 10^{-14}$  cm<sup>2</sup>),  $M$  the atomic mass,  $k_B$  the Boltzmann's constant, and  $T$  the temperature [25]. Assuming a vapour with the number density of atoms of order  $10^{12}$  cm<sup>-3</sup> and with the temperature of about  $10^{-3}$  K (which implies negligible Doppler broadening), the atomic collision rate is as low as 2 Hz. Moreover, with a sufficiently large energy difference between states  $|g\rangle$  and  $|s\rangle$  a single collision is not likely to affect the qubit.

*Conclusions.* In conclusion, we presented a new scheme for cluster state generation based on atomic ensembles and the dipole blockade mechanism. The protocol consists of single-photon sources, atomic ensembles, and realistic photodetectors. The protocol generates GHZ states with probability  $p \sim \eta^{Q/2}$ , where  $Q$  is the number of the qubits, and high fidelity  $F \cong 0.982$ . The protocol is more efficient than any previously proposed probabilistic scheme with realistic photodetectors and sources. In general, number-resolution photodetectors are not required.

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